MOCK TEST – I (JEE MAIN)

ANSWER KEYS, HINTS & SOLUTIONS

PHYSICS

1. **B**

$\vec{A}\times\vec{B}$ is a vector perpendicular to both $\vec{A}$ and $\vec{B}$

Now, $\vec{A}\times\vec{B} = (\hat{i} - 2\hat{j} + \hat{k})\times(3\hat{i} + \hat{j} - 2\hat{k})$

$3\hat{i} + 5\hat{j} + 7\hat{k}$

Now, $\hat{n} = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

$= \dfrac{3\hat{i} + 5\hat{j} + 7\hat{k}}{\sqrt{3^2 + 5^2 + 7^2}}$

$= \dfrac{3\hat{i} + 5\hat{j} + 7\hat{k}}{\sqrt{83}}$

2. **B**

Horizontal velocity remains same

$30^\circ \cos 60^\circ = v \cos 45^\circ$

$\Rightarrow v = 15\sqrt{2}$ m/s

3. **A**

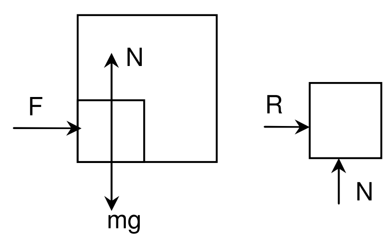
$v = \sqrt{29}\ t$

$a = \dfrac{dv}{dt} = \sqrt{29}$

$R = ma = \sqrt{29}$ N

$F_{net} = \sqrt{N^2 + R^2}$

$= \sqrt{(10)^2 + (29)} = \sqrt{129}$ N



4. **C**

Δp after 1st impact $= ep - (-p)$

$= p(1+e)$

Similarly Δp after 2nd impact $= ep(1+e)$

So, $p_{net} = p(1+e)\left[1 + e + e^2 + ...\right]$

$= \dfrac{p(1+e)}{(1-e)}$

5. **B**

Force of explosion is internal and system is initially at rest

Let the velocities of the first two fragments are $v\hat{i}$ & $v\hat{j}$ and that of the fragment 2m be $v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$,

So, $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$

$\Rightarrow mv\hat{i} + mv\hat{j} + 2m(v_1\hat{i} + v_2\hat{j} + v_3\hat{k}) = 0$

$\Rightarrow v_1 = -\dfrac{v}{2},\ v_2 = -\dfrac{v}{2}$ and $v_3 = 0$

$$\vec{p}_3 = -2m\left(\frac{v}{2}\hat{i} + \frac{v}{2}\hat{j}\right)$$

So, $K_f = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)\left(v_1^2 + v_2^2 + v_3^2\right)$

$= \frac{3}{2}mv^2$

Energy released in explosion

$\Delta E = K_f - K_i$

$= \frac{3}{2}mv^2 - 0 = \frac{3}{2}mv^2$

6. **D**

Angular momentum about point of contact will be conserved

$$L_i = L_f$$

$$\left(\frac{2}{5}Mr^2\right)\frac{v_0}{2r} + Mrv_0 = \left(\frac{2}{5}Mr^2\right)\frac{v}{r} + Mrv$$

$\Rightarrow v = \frac{6}{7}v_0$

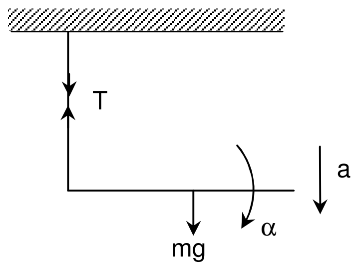
7. **C**

$mg - T = ma$ and

$\frac{mgl}{2} = I\alpha = \frac{ml^2}{3} \times \frac{a}{\left(\frac{l}{2}\right)}$

$\Rightarrow a = \frac{3g}{4}$

So, $T = \frac{mg}{4}$



8. **C**

$I = -\frac{K}{r} = -\frac{dV}{dr}$

$\Rightarrow \int_{V_0}^{V} dV = K\int_{r_0}^{r} \frac{dr}{r}$

$\Rightarrow V = K\ln\frac{r}{r_0} + V_0$

9. **D**

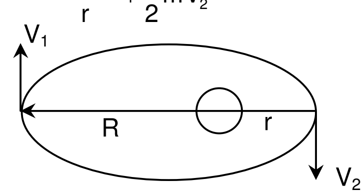
By conservation of angular momentum, $mv_1R = mv_2r$

$\Rightarrow \quad v_1R = v_2r \quad \ldots.(i)$

By conservation of energy, $-\frac{GMm}{R} + \frac{1}{2}mV_1^2 = -\frac{GMm}{r} + \frac{1}{2}mV_2^2$

$\Rightarrow V_1 = \sqrt{\frac{2GMr}{R(R+r)}}$

Now, $mv_1R = m\sqrt{\frac{2GMrR}{r+R}}$



10. **D**

By equation of continuity

$$Av = av'$$

$\Rightarrow v' = \left(\frac{A}{a}\right)v = \left(\frac{\pi R^2}{\pi r^2}\right)\times v$

$\Rightarrow v' = 400$ cm/sec.

11. **D**

For cylinder A ;

$\Delta Q = nC_p\Delta T_1$

For cylinder B, $\Delta Q = nC_v\Delta T_2$

Hence, $nC_p\Delta T_1 = nC_v\Delta T_2$

$(C_v + R)30 = C_v\Delta T_2$

For diatomic gas,

$C_v = \frac{5}{2}R$

$\Rightarrow \Delta T_2 = 42$ K

12. **B**

$\frac{dT}{dt} \propto \Delta\theta \qquad \frac{d\theta}{dt} \propto (\theta - \theta_0)$

$\Rightarrow dT = K\Delta\theta dt$

In first case,

$dT = 61 - 59 = 2^\circ C$; $\Delta\theta = 30^\circ C$, dt = 4 min.

For second case, dT = 2^oC; $\Delta\theta = 20^\circ C$

$K = \frac{dT}{\Delta\theta dt} = \frac{2}{30\times 4} = \frac{1}{60}$

$dt = \frac{dT}{K\Delta\theta} = \frac{2}{\frac{1}{60}\times 20} = 6$ min.

13. **C**

$y = a(\sin\omega t + \cos\omega t)$

$= a\sqrt{2}\left(\sin\omega t\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cos\omega t\right)$

$= a\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)$

The motion is SHM with amplitude $a\sqrt{2}$

14. **B**

$\frac{aT}{x} = \frac{\omega^2 xT}{x} = \frac{4\pi^2}{T^2}\times T$

$= \frac{4\pi^2}{T} = \text{Constt.}$

15. **A**

Relative velocity is $(v + v_0)$

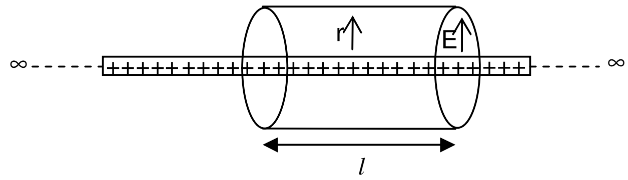
Opponent frequency is $\frac{(v + v_0)}{\lambda}$

The no. of positive crests striking per sec. is same as frequency.

In 3 sec. $3\left(\frac{v + v_0}{\lambda}\right)$

16. **B**

From symmetry, $\vec{E}$ due to a uniform linear charged can only be radially directed. As a Gaussian surface, we can choose a circular cylinder of radius r and length. L, closed at each end by plane caps normal to the axis.



$\varepsilon_o \oint \vec{E}.\,d\vec{s} = q_{in}$

$\varepsilon_o \left[\int \vec{E}.d\vec{s} + \int \vec{E}.d\vec{s} \right] = q_{in}$

Cylindrical Plane Surface

$\varepsilon_o E(2\pi r l) + E.ds \ \cos 90^\circ = \lambda l$

$E = \dfrac{\lambda l}{\varepsilon_o 2\pi r l} = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

The direction of $\vec{E}$ is radially outwards for a line of positive charge.

17. **B**

$E_x = -\dfrac{\partial V}{\partial x} = -(10x + 5y) = -10 + 10 = 0$

$E_y = -\dfrac{\partial V}{\partial y} = -5x = -5$

$\therefore \vec{E} = -5\hat{j}$ V/m.

18. **A**

Work done = change in potential energy

$= U_2 - U_1$

$U_1 = (1/2)E^2C$

$U_2 = \dfrac{1}{2}\dfrac{(EC)^2}{C'} = \dfrac{1}{2}\dfrac{E^2C^2}{C}\varepsilon_r = (1/2)E^2C\varepsilon_r$

$\therefore$ Work done = $(1/2)E^2C(\varepsilon_r - 1)$.

19. **D**

The resistance of the parallel combination of 2Ω and 3Ω resistors is given by $\dfrac{1}{R} = \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{5}{6} \Rightarrow R = 1.2\Omega$

This resistance is in series with 2.8Ω giving a total effective resistance $= 1.2 + 2.8\Omega = 4\Omega$.

In the steady state, charge on the capacitor C has stablised and hence no current passes through 4Ω resistor which is in series with the capacitor.

Thus the current through the circuit = 6/4 = 1.5 A,

$V_{AB} = 1.5 \times 1.2 = 1.8V$, I through 2Ω resistor = 1.8/2 = 0.9 A.

20. **A**

$r = 1.5\Omega,\ l_1 = 52cm,\ l_2 = 40cm$

$\dfrac{r}{R} = \dfrac{l_1 - l_2}{l_2}$

resistance R = $\dfrac{r l_2}{(l_1 - l_2)}$

$= \dfrac{1.5 \times 40}{(52 - 40)}$

$= \dfrac{1.5 \times 40}{12} = 5\Omega$

21. **D**

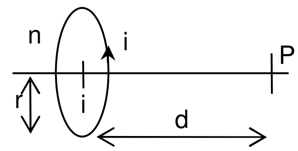
$$R = \frac{\rho l}{A} = \frac{\rho l^2}{Al} = \frac{\rho l^2}{V} = \frac{\rho l^2}{m/d} = \frac{\rho d l^2}{m}$$

or $R \propto \frac{l^2}{m}$

$$R_1 : R_2 : R_3 = \frac{l_1^2}{m_1} : \frac{l_2^2}{m_2} : \frac{l_3^2}{m_3} = \frac{25}{1} : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$$

22. **A**

The magnetic field at P due to the flat coil of n turns, radius r, carrying current i is



$$B = \frac{\mu_0}{2} \cdot \frac{nir^2}{(d^2 + r^2)^{3/2}} \cong \frac{\mu_0}{2} \cdot \frac{nir^2}{d^3} (d >> r) = \frac{\mu_0}{2\pi} \cdot \frac{n(\pi r^2) i}{d^3} = \frac{\mu_0}{2\pi} \cdot \frac{\mu}{d^3}.$$

23. **C**

$B = \frac{\mu_0 Ni}{l}$ where N = Total number of turns, l = length of the solenoid

$$\Rightarrow 0.2 = \frac{4\pi \times 10^{-7} \times N \times 10}{0.8}$$

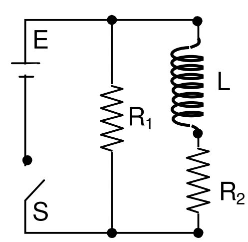
$$\Rightarrow N = \frac{4 \times 10^4}{\pi}$$

Since N turns are made from the winding wire so length of the wire $(L) = 2\pi r \times N [2\pi r = \text{ length of each turns}]$

$$\Rightarrow L = 2\pi \times 3 \times 10^{-2} \times \frac{4 \times 10^4}{\pi} = 2.4 \times 10^3 m$$

24. **D**

$$I_1 = \frac{E}{R_1} = \frac{12}{2} = 6A$$



$$E = L\frac{dI_2}{dt} + R_2 \times I_2$$

$$I_2 - I_0 \left(1 - e^{-t/t_0}\right) \Rightarrow I_0 = \frac{E}{R_2} = \frac{12}{2} = 6A$$

$$t_0 = \frac{L}{R} = \frac{400 \times 10^{-3}}{2} 0.2$$

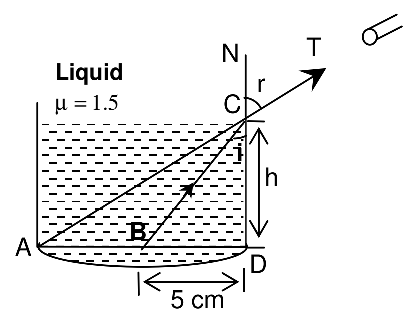
$$I_2 = 6\left(1 - e^{-t/0.2}\right)$$

Potential drop across L = E - $R_2 I_2 = 12 - 2 \times 6\left(1 - e^{-bt}\right) = 12\ e^{-5t}$

25. **D**

It is mentioned in the problem that on filling the vessel with the liquid, point B is observed for the same setting; this means that the image of point B, is observed at A, because of refraction of the ray at C. For refraction at C,

$$\frac{\sin r}{\sin i} = {}_a\mu_l = 1.5$$



Now,

$$\sin r = \frac{AD}{AC} = \frac{10}{\sqrt{10^2 + h^2}}$$

Where h is the height of vessel.

$\sin i = \frac{BD}{BC} = \frac{5}{\sqrt{5^2 + h^2}}$

$\therefore \frac{10}{\sqrt{100 + h^2}} \cdot \frac{\sqrt{25 + h^2}}{5} = 1.5$

$\therefore \frac{25 + h^2}{100 + h^2} = \frac{9}{16}$

$\therefore 400 + 16h^2 = 900 + 9\,h^2$

$\therefore 7h^2 = 500$

$\therefore h = 8.45$ cm

26. **C**

In this case

$\frac{1}{f_L} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{(\mu - 1)}{R}$

and $F_M = \frac{(-R)}{2}$

so $P_L = \frac{1}{f_L} = \frac{(\mu - 1)}{R}$

and $P_M = -\frac{1}{f_M} = \frac{2}{R}$

and hence power of system

$P = P_L + P_M + P_L = 2P_L + P_M$

$\Rightarrow P = \frac{2(\mu - 1)}{R} + \frac{2}{R} = \frac{2\mu}{R}$

$\therefore F = -\frac{1}{P} = -\frac{R}{2\mu}$

i.e. the lens will be equivalent to a converging mirror of focal length $(R/2\mu)$

27. **A**

For eyepiece,

$v_E = -25$ cm, $f_E = +5$ cm

$\Rightarrow u_E = -4.17 \text{ cm} \simeq -4.2 \text{ cm}$

$L = v_0 + |u_E| = 12.2$ cm here $v_0 = 8$ cm

$f_0 = +1 \text{ cm} \Rightarrow u_0 = -1.1 \text{ cm}$

28. **B**

Wavelength associated with a particle is given by

$\lambda = \frac{h}{\sqrt{2mE}}$

$\lambda = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{150}{V}} A^\circ$ (for electron M = 9.11×10^{-31}kg)

$\lambda = \sqrt{\frac{150}{10 \times 10^3}} = 0.122\ A^\circ$

For minima

$d \sin\theta = n\,\lambda$

For first minima n = 1

$0.55 \sin\theta = 0.122$

$\sin\theta = \frac{0.122}{0.55} = 0.2218$

29. **D**

$h\nu = h\nu_0 + E$

$E = h\nu - h\nu_0$

But $\nu = \frac{c}{\lambda}$

$E = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = hc\left(\frac{\lambda_0 - \lambda}{\lambda_0 \lambda}\right)$

30. **D**

The resistivity of pure Si is given by

$\rho = \frac{1}{\sigma} = \frac{1}{e(n_e\mu_e + n_h\mu_h)} = \frac{1}{en_1(\mu_e + \mu_h)}$

or $n_i = \frac{1}{e\rho(\mu_e + \mu_h)} = \frac{1}{1.6\times10^{-19}\times3000(0.12+0.045)}$

$= 1.26\times10^{16}\,m^{-3}$

When 10^{19} atoms of phosphorous (donor atoms of valence five) are added per m^3, the semiconductor becomes n – type semiconductor.

$\therefore n_e - n_h \approx n_e = N_d = 10^{19} \quad \because n_h = 1.26\times10^{16}$

Resitivity $\rho = \frac{1}{n_e\mu_e e} = \frac{1}{1.6\times10^{-19}\times10^{19}\times0.12} = 5.21\ \Omega m$

CHEMISTRY

1. **A**

HF has high melting point than HCl due to intermolecular hydrogen bonding while HCl < HBr < HI (melting point) due to increase in molecular weight.

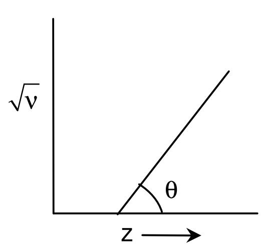
2. **C**

The room energy 3/2RT derives from kinetic (moment) energy of a gas or material.

3. **A**

4. **A**

5. **B**



$\sqrt{\nu} = a(z-b)$

Now, $\tan 45° = 1 = a$

ab = 1

$\sqrt{\nu} = 39 - 1 = 38$

$\nu = 1444\ s^{-1}$

6. **A**

$\frac{T_b}{T_c} \simeq \frac{2}{3}$

7. **C**

Volume $\left(a^3\right) = \left(5\times10^{-8}\,cm\right)^3 = 1.25\times10^{-22}\,cm^3$

d = 4g cm^{-3}

Mass of unit cell = 5 x 10^{-22} g

Mass of one molecule $= \frac{72g}{6.023\times10^{23}\,mole^{-1}}$ = 1.195 x 10^{-22} g

No. of FeO molecules per unit cell $= \frac{5\times10^{-22}}{1.195\times10^{-22}} = 4$

The $4Fe^{+2} + 4O^{-2}$

8. **D**

The decrease is 'S' character between bond angle 120 and 109.5 is 8.3%

For 2.5^o decrease the decrease in 'S' character

$= \frac{8.3\times2.5}{10.5} = 1.98$

Thus S-character = 25 – 1.98 = 23.02%

9. **D**

Average molar mass $= \frac{m_1x_1 + m_2x_2}{x_1 + x_2} = \frac{28\times80 + 32\times20}{100}$

= 28.8 g mol^{-1} A

Molar volume at STP (V_m) = 24.8 L

Density $= \frac{28.8}{24.8} = 1.161\,gL^{-1}$

10. **A**

11. **D**

The highest rise would be recorded. If neutralization process is complete

$$2NaOH + H_2SO_4 \rightarrow Na_2SO_4 + H_2O$$

Let meq of NaOH x, H_2SO_4 = y, then

x = y for complete neutralization

$N\times V_{NaOH} = N\times V_{H_2SO_4}$

$\frac{V_{NaOH}}{V_{H_2SO_4}} = \frac{1}{1}$ then 50, 50 ml each

12. **C**

13. **B**

$\therefore t \propto (Ao)^{1-n}$

or $const\times(Ao)^{n-1} = t$

or $t\times(Ao)^{1-n} = const$

or $\sqrt{t\times(Ao)^{n-1}} = const$

or $\sqrt{t}\times(Ao)^{\frac{n-1}{2}} = const$

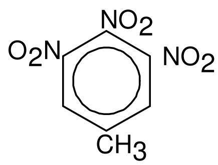
but from question

$\frac{n-1}{2} = 1$, n = 3

14. **C**

Because elimination of 'H' is easier that of 'D' and this type of elimination takes place in trans manner.

15. **A**



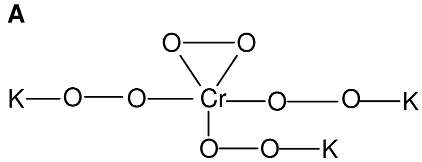
$\xrightarrow{\text{Explosion}} CO + N_2 + H_2O + CO_2$

16. **D**

17. **C**
Ag being salt is alloyed with copper. The composition of a silver alloy in expressed as its fineness. The amount of Ag in 1,000 parts of the alloy hence. 82.5% Ag and 17.5% Cu

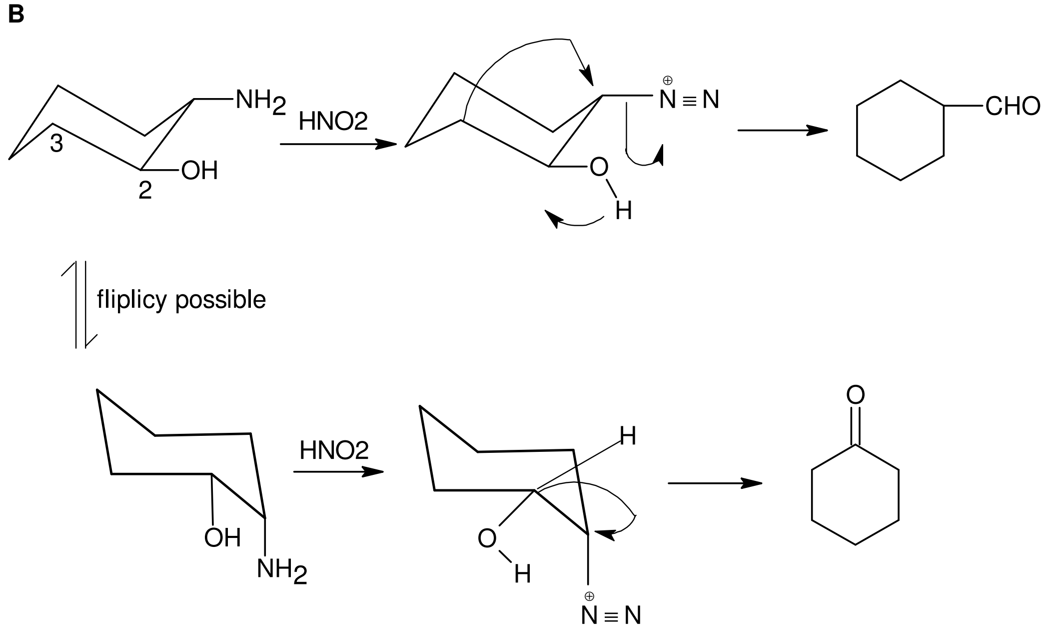
18. **C**
$2As_2S_3 + 4NaOH \rightarrow NaAsO_2 + 3NaAsS_2 + 2H_2O$ soluble

19. **A**



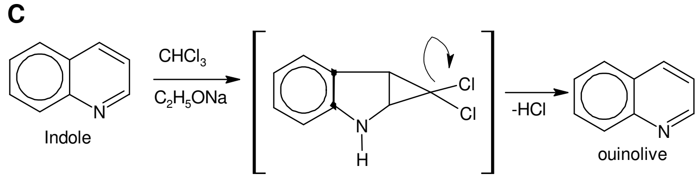
20. **C**
Follow structure of one dimensional silicates

21. **B**



Ring bond between C_2 and C_3 as well as the Hydrogen atom can migrate independently to form cyclopentane carboxaldehycle and cyclohexananone respectively.

22. **C**



23. **C**

24. **C**

25. **A**

Urotropine $(CH_2)_6 N_4$ hexamethyl diamine = $NH_2(CH_2)_6 - NH_2$

26. **B**

27. **A**

(i) K_2PtCl_6 – three ions

(ii) $\left[Pt(NH_3)_2 Cl_4\right]$ - one molecule

(iii) $\left[Pt(NH_3)_3 Cl_3\right]Cl$ - Two ions

(iv) $\left[Pt(NH_3)_5 Cl\right]Cl_3$ - four ions

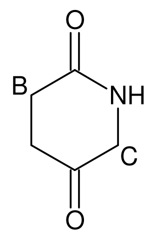
and conductivity $\propto$ no. of ions

28. **A**

At isoelectric point migration of α - amino acid not observed

29. **A**

$RCH_2OH \rightarrow \underset{(A)}{RCHO} + HBr +$



$A \xrightarrow[I_2]{NaOH} CHI_3$ (R – should be CH_3 when it shows haloform)

30. **B**

According to bent rule.

MATHEMATICS

1. **C**

Let $n = 10^4 x_1 + 10^3 x_2 + 10^2 x_3 + 10x_4 + x_5$

$m = 10^3 x_1 + 10^2 x_2 + 10x_4 + x_5$

$\therefore \ 10m - n = 10^2 (x_4 - x_3) + 10(x_5 - x_4) - x_5$

$10m - n$ is a three digit number & $\frac{10m-n}{m}$ is an integer where numerator is three digit & denominator is four digit.

$\therefore \ 10m - n = 0 \ \Rightarrow x_3 = x_4 = x_5 = 0$

$\therefore \ n = 10^4 x_1 + 10^3 x_2 = 10^3 (10x_1 + x_2)$

$\therefore \ 10x_1 + x_2$ is a two digit no.

$\therefore \ 10 \le 10x_1 + x_2 \le 99$

2. **B**

$a_{n+2} - 1 = a_1 a_2 \ .... a_{n+1}$

it follows that $a_{n+1} = \frac{a_{n+2} - 1}{a_{n+1} - 1}$

$$\therefore \ \frac{1}{a_{n+2} - 1} = \frac{1}{(a_{n+1} - 1)a_{n+1}} = \frac{1}{a_{n+1} - 1} - \frac{1}{a_{n+1}}$$

$$1 + \sum_{n=1}^{\infty} \frac{1}{a_{n+1}} = 1 + \sum_{n=1}^{\infty} \left(\frac{1}{a_{n+1} - 1} - \frac{1}{a_{n+2} - 1} \right)$$

$$= 1 + \frac{1}{a_2 - 1} = 2$$

3. **C**

$\bar{a}\,.\,\bar{c} = \bar{b}\,.\,\bar{c} = \cos\theta$

Taking dot product with $\bar{a}$, $\cos\theta = \alpha$

Taking dot product with $\bar{b}$, $\cos\theta = \beta$

Taking dot product with $\bar{c}$

$1 = \alpha\cos\theta + \beta\cos\theta + \gamma\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

$$\left[\bar{a}\ \bar{b}\ \bar{c}\right]^2 = \begin{vmatrix} 1 & 0 & \cos\theta \\ 0 & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix} = 1 - 2\cos^2\theta$$

$\therefore$ So, $1 = 2\cos^2\theta + \gamma\sqrt{1-2\cos^2\theta}$

$\gamma = \sqrt{1-2\cos^2\theta}$

So, $\alpha^2 + \beta^2 + \gamma^2 = 1$

4. **B**

$5^{\left(\frac{x}{\sqrt{x}+2} - \frac{4}{\sqrt{x}+2}\right)\frac{1}{x-4}} = 5^{3-2\left(\frac{x-2}{x-4}\right)}$

$5^{\frac{1}{\sqrt{x}+2}} = 5^{\frac{x-8}{x-4}}$

$\dfrac{1}{\sqrt{x}+2} = \dfrac{x-8}{x-4}$

$\sqrt{x} - 2 = x - 8$

$x - \sqrt{x} - 6 = 0$

Let $\sqrt{x} = t$

$t^2 - t - 6 = 0$

$t^2 - 3t - 2t - 6 = 0$

$(t-3)(t+2) = 0$

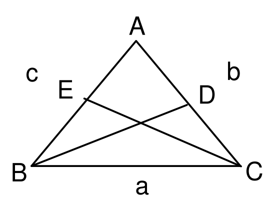
$t = 3,\ -2$

$\sqrt{x} = 3$

$x = 9$

5. **A**

$\dfrac{a}{R} = 2,\ \dfrac{b}{R} = 1.5$



$\dfrac{2R\sin A}{R} = 2$

$\Rightarrow \sin A = 1$

$\therefore\ c = \sqrt{a^2 - b^2} = R\sqrt{\dfrac{7}{2}}$

$BD = \sqrt{(AB)^2 + (AD)^2}$

$AD = (AC)\dfrac{AB}{AB+AC} = \dfrac{bc}{a+c}$

$AE = \dfrac{bc}{a+b}$

$AD = \dfrac{3R\sqrt{7}}{2\left(4+\sqrt{7}\right)},\ AE = \dfrac{3R\sqrt{7}}{14}$

$BD = \dfrac{2R\sqrt{7}}{\sqrt{7}+1},\ CE = \dfrac{3R\sqrt{2}}{\sqrt{7}}$

6. **B**

1^{st} term of A.P. & G.P. = a

Let common ratio be r & common difference be d.

$$a+2d=ar^2 \qquad \ldots\ldots(1)$$

$$\& \ a+d=ar+0.25 \qquad \ldots\ldots(2)$$

From (1) & (2), $ar^2-2ar+a-0.5=0$

$\therefore \quad 4a^2-4a(a-0.5)\geq 0$

$2a \geq 0$

$0<\dfrac{2a}{2a}<2 \quad \Rightarrow a\in R$

$a(a-0.5)>0$ & $a(4a-4a+a-0.5)>0$

$a\in(-\infty,\ 0)\cup\left(\dfrac{1}{2},\ \infty\right)$

7. **C**

Let the subsets be n words from the alphabet {0, 1}. Let a_n be the no. of binary words with no two successive ones. The words can start either with 0 and may continue in a_{n-1} ways or they start with 10 & may continue in a_{n-2} ways

$a_n=a_{n-1}+a_{n-2} \qquad a_1=2,\ a_2=3$

$a_3=5\ a_4=8\ a_5=13\ a_6=21\ a_7=34\ a_8=55,\ a_9=89$

8. **A**

$f(x)=\dfrac{1}{x}$

$$I=\int_2^3\frac{\frac{3}{x^5}-\frac{1}{x}}{1-\frac{1}{x^4}}dx$$

$$=\int_2^3\frac{\frac{3}{x^7}-\frac{1}{x^3}}{\frac{1}{\frac{1}{x^2}-\frac{1}{x^6}}}dx$$

Let $\dfrac{1}{\frac{1}{x^2}-\frac{1}{x^6}}=z$

$$\therefore \quad I=\frac{1}{2}\int_{\frac{15}{64}}^{\frac{80}{729}}\frac{dz}{z}$$

$$=\frac{1}{2}[\log z]_{15/64}^{80/729}$$

$$=\frac{1}{2}\log\left(\frac{2^{10}}{3^7}\right)$$

$$=\alpha=10,\ \beta=7$$

9. **D**

$\cos^2 C == \cos^2 B - \sin^2 A = \cos(B+A)\cos(B-A)$

$\Rightarrow \cos C\,.\,\cos\left(\pi - \overline{A+B}\right) = \cos(\pi - C)\cos(B-A)$

$\Rightarrow \cos C\{-\cos(A+B)\} = -\cos C\cos(B-A)$

$\Rightarrow \cos C\left[-\cos(A+B)+\cos(B-A)\right] = 0$

$\Rightarrow \cos C = 0$ or $2\sin B\sin A = 0$ (Not possible)

$\Rightarrow C = 90^o$

$P = \sin A\sin(90-A)+0+0$

$= \frac{1}{2}\sin 2A \quad \Rightarrow 0 < P \le \frac{1}{2}$

10. **B**

$2 - e^{x^2} < 1$ (if $x \to 0$)

$e^{x^2} + 1 > 2$

$\frac{2-e^{x^2}}{e^{x^2}+1} < 1 \quad \Rightarrow 0 < \frac{2-e^{x^2}}{e^{x^2}+1} < 1$ if $x \to 0$

$\frac{x}{\tan x} < 1$

11. **A**

Differentiating with respect to x

$f(x) + xf'(x) - f(x) = 1 + \frac{1}{\sqrt{x^2+1}-x}\left(\frac{x}{\sqrt{x^2+1}} - 1\right)$

$xf'(x) = 1 - \frac{1}{\sqrt{x^2+1}}$

$\int_0^1 g(x)dx = \int_0^1 \left(1 - \frac{1}{\sqrt{x^2+1}}\right)dx$

12. **C**

$(a_1+a_2+a_3+a_4)^4 \le 256\ a_1a_2a_3a_4$

$\Rightarrow \frac{a+a_2+a_3+a_4}{4} \le (a_1a_2a_3a_4)^{\frac{1}{4}}$

$\Rightarrow$ A.M. $\le$ G.M.

But A.M. $\ge$ G.M. $\Rightarrow$ A.M. $=$ G.M.

$\therefore\ a_1 = a_2 = a_3 = a_4$

13. **B**

$I = 10\int_0^{\pi} \frac{\cos 6x\cos 7x\cos 8x\cos 9x}{1+e^{2\sin^3 4x}}\,dx$

$2I - 10\int_0^{\pi} \cos 6x\cos 7x\cos 8x\cos 9x\ dx \quad \left(\because\ \int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx\right)$

$I = 5\int_0^{\frac{\pi}{2}} \cos 6x\cos 7x(\cos 16x + \cos 2x)dx$

$= 5\int_0^{\frac{\pi}{2}} \cos 6x\cos 7x\cos 2x\ dx + 0$

$$I = 10\int_0^{\frac{\pi}{4}} \cos 6x \cos 7x \cos 2x \, dx$$

14. **D**

Let $\cot\frac{A}{2} = x,\ \cot\frac{B}{2} = y,\ \cot\frac{C}{2} = z$

$$x + y + z = \frac{3s - (a+b+c)}{r} = \frac{s}{r}$$

$$\therefore\ x^2 + (2y)^2 + (3z)^2 = \left[\frac{6}{7}(x+y+z)\right]^2$$

$$\Rightarrow 13x^2 + 160y^2 + 405z^2 - 72(xy + yz + zx) = 0$$

$$\Rightarrow (3x - 12y)^2 + (4y - 9z)^2 + (18z - 2x)^2 = 0$$

$$\therefore\ \frac{x}{y} + \frac{y}{z} + \frac{z}{x} = 4 + \frac{9}{4} + \frac{1}{9}$$

15. **B**

$$\frac{1}{t_r} = \frac{1}{r+2} - \frac{1}{r+3}$$

$$\therefore \sum_{r=1}^{671}\frac{1}{t_r} = \sum_{r=1}^{671}\left(\frac{1}{r+2} - \frac{1}{r+3}\right) = \left[\frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + ... + \frac{1}{670} - \frac{1}{671}\right]$$

$$= \frac{1}{3} - \frac{1}{671} = \frac{668}{2013}$$

$$\Rightarrow \lambda = 2014$$

16. **C**

$$(z^3 + 1) + 2z(z+1) = 0$$

$$(z+1)(z^2 - z + 1 + 2z) = 0$$

$$\Rightarrow z = -1,\ \omega,\ \omega^2$$

$$f(z) = z^{2014} + z^{1007} + 1$$

$$f(-1) \neq 0 \text{ but } f(\omega) = 0 = f(\omega^2)$$

$\therefore \omega$ and ω^2 are two common roots.

17. **A**

Put x = y = 0 $\quad \Rightarrow f(0) = 0$

diff. w.r.t. x keeping y constant

$$f'(x+y) = f'(x) + e^{x+y} + (x+y)e^{x+y} - xe^x - e^x + 2y$$

Replace y by x & x by 0, then

$$f'(x) = 1 + e^x + xe^x - 1 + 2x$$

$$= e^x + xe^x + 2x$$

$$\therefore\ \int_0^1 f'(x)e^{f(x)}dx = \left[e^{f(x)}\right]_0^1 = e^{f(1)} - e^{f(0)}$$

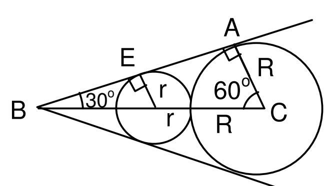
18. **B**

BD = 2r

BC = 2R

$\therefore$ 2r + r + R = 2R

R = 3r



$\Rightarrow r = \frac{R}{3}$

19. **B**

On simplifying $f(x) = 3$

$\therefore \quad g(x) = x^3 - 3x + 1$

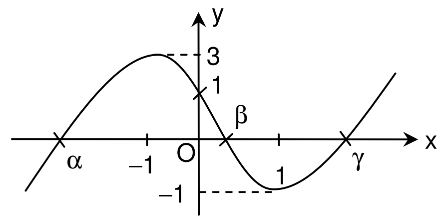
$g(g(x)) = 0$ when $g(x) = \alpha, \beta, \gamma$

Where α, β, γ are roots of $g(x) = 0$

$g(x) = \alpha$ has 1 solution

$g(x) = \beta$ has 2 solution

$g(x) = \gamma$ has 3 solution



20. **A**

Let any point of $xy = 4$ be $R\left(2t, \frac{2}{t}\right)$

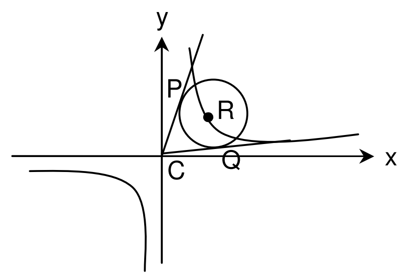
Let circumcentre of ΔCPQ by (h, k) which is midpoint of CR

$\frac{2t+0}{2} = h$ & $\frac{2}{2t} = k$

$\therefore \quad t = h \qquad \frac{1}{t} = k$

$\Rightarrow hk=1$

$\Rightarrow xy=1$



21. **A**

$P(x) = (x-a_1)(x-a_3)(x-a_5)(x-a_7) + (x-a_2)(x-a_4)(x-a_6)(x-a_8)$

$P(a_1) = +ve$

$P(a_2) = -ve$

$P(a_3) = -ve$

$P(a_4) = +ve$

$P(a_5) = +ve$

$P(a_6) = -ve$

$P(a_7) = -ve$

$P(a_8) = +ve$

Hence, $P(x) = 0$ has all positive real roots.

22. **D**

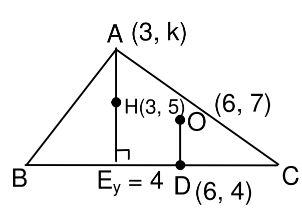
Let x + 1 = t in 1st integral

$$\int_1^e \frac{e^{\frac{t^2-2}{2}}}{t}dt + \int_1^e t\ln t \,.\, e^{\frac{t^2-2}{2}}\, dt = \int_1^e \frac{e^{\frac{t^2-2}{2}}}{t}dt + \left[\ln t e^{\frac{t^2-2}{2}}\right]_1^e - \int_1^e \frac{e^{\frac{t^2-2}{2}}}{t}dt = e^{\frac{e^2-2}{2}}$$

23. **C**

$G = \left(\frac{12+3}{3}, \frac{14+5}{3}\right)$

$\left(5, \frac{19}{3}\right)$



Equation of AD $\quad y-4=\dfrac{\frac{19}{3}-4}{5-6}(x-6)$

$$\Rightarrow y-4=\frac{-7}{3}(x-6)$$

point A(3, k) satisfies this

$k-4=\dfrac{-7}{3}(-3)$

$k=7+4=11$

$\therefore$ Circumradius $=\sqrt{(6-3)^2+(7-1)^2}=\sqrt{9+16}=5$

$\therefore$ Area of circumcircle $=25\pi$

24. **B**

$AA^T=I \quad \Rightarrow |A|=\pm 1$

$|\text{adj}(\text{adj } A)|=|A|^{(n-1)^2}=\pm 1$

25. **C**

$f(x)=e^{\lim_{n\to\infty}\left(\cos\sqrt{\frac{x}{n}}-1\right)n}$

$=e^{-\lim_{n\to\infty}\frac{2\sin^2\left(\frac{1}{2}\sqrt{\frac{x}{n}}\right)}{\frac{1}{n}}}$

$=e^{-\frac{x}{2}}$

$g(x)=e^{\lim_{n\to\infty}\frac{x\left(e^{\frac{1}{n}}-1\right)}{\frac{1}{n}}}=e^x$

$g^{-1}(x)=\ln x$

$f^{-1}(x)=2\ln\left(\frac{1}{x}\right);\ 0<x\le 1$

$g^{-1}\left(f^{-1}(x)\right)=\ln\left(2\ln\left(\frac{1}{x}\right)\right)$ for $0<x<1$

$\therefore$ Domain of $h(x)$ is (0, 1)

26. **A**

$\cos^3 x+\dfrac{\cos x}{4}\ge \cos^2 x$

$\cos A,\ \cos B\ \&\ \cos C$ are nonnegative

$x_1+x_3\ge \cos^2 A+\cos^2 B+\cos^2 C+2\cos A\cos B\cos C=2x_2$

$\Rightarrow x_1+x_2+x_3\ge 3x_2=\dfrac{3}{2}$

$\because\ \cos^2 A+\cos^2 B+\cos^2 C+2\cos A\cos B\cos C=1$

27. **C**

$^{12}C_4\times {}^8C_4\times {}^4C_4=\dfrac{12!}{(4!)^3}$

28. **D**

S is not symmetric & T is reflexive, symmetric & transitive.

29. **B**

Let $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

where x, y, z are positive integers

$\therefore \quad x + y + z \le 12$

The no. of values of $\bar{r}$ is no. of positive integral solutions of $x + y + z \le 12 = \sum_{n=3}^{12} {}^{n-1}C_2 = {}^{12}C_9$

30. **D**

$f_1(1)$ & $f_1(3)$ are of opposite sign so one root lies in between 1 & 3. By graph $f_1(x)$ has min. 2 roots

By graph $f_2(x)$ has min. 3 roots so $f'(x)$ has 2 roots by Rolle's theorem.

Let $g(x) = f_1(x) \cdot f_2'(x)$ has min. 4 roots

$\Rightarrow \quad g'(x)$ has min. 3 roots